

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2024**THIRD YEAR [BATCH 2022-25]****MATHEMATICS [HONOURS]**

Date : 10/12/2024

Time : 11.00 am – 1.00 pm

Paper : DSE (Probability Theory and Statistics)

Full Marks : 50

Answer all the questions. Maximum you can score 30 from **group A** and 20 from **group B**.

Group : A

1. If a die is thrown k times, show that the probability of an even number of sixes is

$$\frac{\left(1 + \left(\frac{2}{3}\right)^k\right)}{2}$$

[6]

2. A random variable X has the continuous distribution with probability density function

$$f(x) = kx^2(1-x^2)^3 \text{ if } -1 < x < 1 \\ = 0 \text{ otherwise.}$$

Evaluate k and construct the distribution function. What is $P\left[\frac{1}{3} < X < \frac{5}{4}\right]$ [2+3+1]

3. If the random variable X follows standard normal distribution, show that $\frac{1}{2}X^2$ follows gamma distribution with parameter $\frac{1}{2}$. [6]

4. Let X be a random variable uniformly distributed over the interval $[0, 2\pi]$. Let $V = \cos X$ and $U = \sin X$. Check whether $E(UV) = E(U)E(V)$. Are U and V independent? [5+1]

5. For two random variables X and Y , with $E(X) = E(Y)$, the regression lines $y = ax + b$ and $x = \alpha y + \beta$, show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$. [6]

6. Let X be a continuous random variable with mean m_X and variance σ_X^2 . Show that for every $\varepsilon > 0$, $P(|X - m_X| \geq \varepsilon) \leq \frac{\sigma_X^2}{\varepsilon^2}$. [6]

Group : B

7. A population is defined by the density function.

$$f(x; \alpha) = \frac{x^{l-1} e^{-x/\alpha}}{\Gamma(l) \alpha^l} \quad 0 < x < \infty, \alpha > 0, \text{ where } l \text{ is a constant. Estimate the parameter } \alpha \text{ by the}$$

method of maximum likelihood and show that the estimator is consistent and unbiased. [4+1+1]

8. Find a confidence interval for mean of normal (m, σ) population, when σ is known. [5]

9. Let p denotes the probability of getting a head, when a given coin is tossed once. Suppose that $H_0 : p = 0.5$ is rejected in favour of $H_1 : p = 0.6$ if 10 trials result in 7 or more heads. Calculate the probabilities of Type 1 and Type-II errors. [2+3]
10. A group of 8 students have scored in a class test as 16,9,8,18,14,10,12,15 out of 20. Test at 5% level of significance whether mean marks of the population be greater than 12, assuming standard deviation of scores of population 2.5. Also discuss the case when σ is not known. Given $t_{0.05;7} = 1.895$. [4+4]

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